1 Boosting with Decision Stump

Consider the following 2-class binary problem

\[ C_{y=-1} = \{(-3,-1), (-3,1), (3,-1), (3,1)\} \]
\[ C_{y=1} = \{(-1,-1), (-1,1), (1,-1), (1,1)\} \]

Using the error and weight update rule of the discrete AdaBoost, answer the following question

a) What are the first 2 decision stumps, and what are their corresponding thresholds.

b) Are these sufficient to obtain perfect classification?
2 Weak Classifiers

Selecting the proper model of weak classifier is often a very important step when tackling new classification problems. Presented below are a number of such problems; for each dataset:

a) Propose a weak learner that will excel in classifying the data

b) Estimate the number of weak learners necessary to perform the task optimally

Figure 1: Checkerboard

Figure 2: Concentric rings

Figure 3: The serpent

Figure 4: Stop sign
3 The AdaBoost error function (to be done at home)

By differentiating the error function

\[ w_i^{(t+1)} = w_i^{(t)} \exp \left( -\frac{1}{2} \alpha_j y_i \varphi_j(x_i) \right). \]  

with respect to \( \alpha_j \), show that the parameters \( \alpha_j \) in the AdaBoost algorithm are updated using

\[ \alpha_j = \ln \left( \frac{1 - \epsilon_j}{\epsilon_j} \right). \]  

in which \( \epsilon_j \) is defined by

\[ \epsilon_j = \frac{\sum_{i=1}^{N} w_i I(\varphi_j(x_i) \neq y_i)}{\sum_{i=1}^{N} w_i}. \]  
